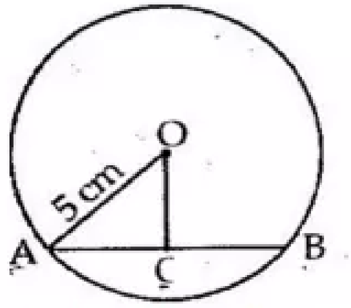
Chapter 17. Circle

Exercise 17(A)

Solution 1:

Let AB be the chord and O be the centre of the circle.
Let OC be the perpendicular drawn from O to AB.



We know, that the perpendicular to a chord, from the centre of a circle, bisects the chord.

$\therefore$ AC = CB = 3 cm

In ΔOCA,

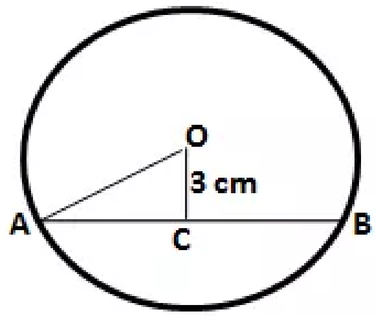
$OA^2 = OC^2 + AC^2$ (By Pythagoras theorem)

$\Rightarrow OC^2 = (5)^2 - (3)^2 = 16$

$\Rightarrow OC = 4$ cm

Solution 2:

Let AB be the chord and O be the centre of the circle.
Let OC be the perpendicular drawn from O to AB.



We know, that the perpendicular to a chord, from the centre of a circle, bisects the chord.

$\therefore$ AB=8 cm

$\Rightarrow AC=CB=\frac{AB}{2}$

$\Rightarrow AC=CB=\frac{8}{2}$

$\Rightarrow$ AC=CB=4 cm

In ΔOCA,

$OA^2 = OC^2 + AC^2$ (By Pythagoras theorem)

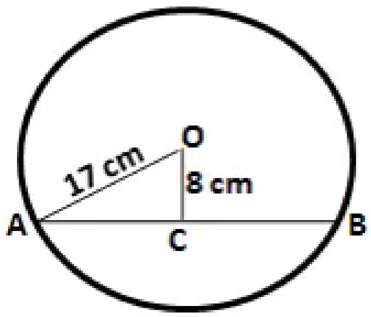
$\Rightarrow OA^2 = (4)^2 + (3)^2 = 25$

$\Rightarrow OA = 5$ cm

Hence, radius of the circle is 5 cm.

Solution 3:

Let AB be the chord and O be the centre of the circle.
Let OC be the perpendicular drawn from O to AB.



We know, that the perpendicular to a chord, from the centre of a circle, bisects the chord.

$\therefore$ AC = CB

In ΔOCA,

$$OA^2 = OC^2 + AC^2 \quad \text{(By Pythagoras theorem)}$$

$$\Rightarrow \quad AC^2 = (17)^2 - (8)^2 = 225$$

$$\Rightarrow \quad AC = 15 \text{ cm}$$

$$\therefore \quad AB = 2\,AC = 2 \times 15 = 30 \text{ cm.}$$

Solution 4:

Let AB be the chord of length 24 cm and O be the centre of the circle.
Let OC be the perpendicular drawn from O to AB.
We know, that the perpendicular to a chord, from the centre of a circle, bisects the chord.

$\therefore$ AC = CB = 12 cm

In ΔOCA,

$$OA^2 = OC^2 + AC^2 \quad \text{(By Pythagoras theorem)}$$

$$= (5)^2 + (12)^2 = 169$$

$$\Rightarrow \quad OA = 13 \text{ cm}$$

$\therefore$ radius of the circle = 13 cm.

Let A'B' be the new chord at a distance of 12 cm from the centre.

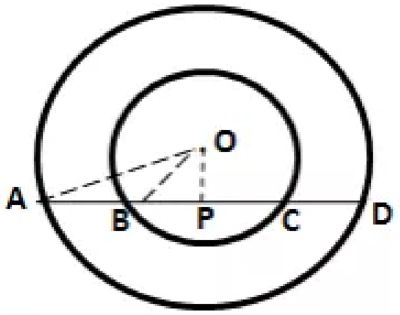
$$\therefore \quad (OA')^2 = (OC')^2 + (A'C')^2$$

$$\Rightarrow \quad (A'C')^2 = (13)^2 - (12)^2 = 25$$

$$\therefore \quad A'C' = 5 \text{ cm}$$

Hence, length of the new chord = $2 \times 5 = 10$ cm.

Solution 5:



For the inner circle, BC is a chord and OP ⊥ BC.

We know that the perpendicular to a chord, from the centre of a circle, bisects the chord.

∴ BP = PC

By Pythagoras Theorem,

$$OB^2 = OP^2 + BP^2$$

$$\Rightarrow \quad BP^2 = (20)^2 - (16)^2 = 144$$

$$\therefore \quad BP = 12 \text{ cm}$$

For the outer circle, AD is the chord and OP ⊥ AD.

We know that the perpendicular to a chord, from the centre of a circle, bisects the chord.

∴ AP = PD

By Pythagoras Theorem,

$OA^2 = OP^2 + AP^2$

=> $AP^2 = (34)^2 - (16)^2 = 900$

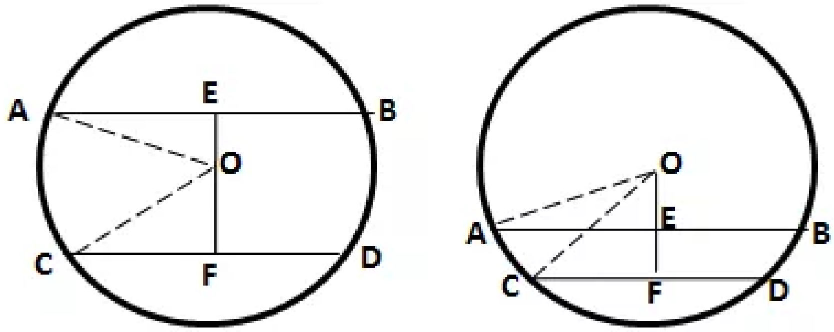
=> AP = 30 cm

AB = AP - BP = 30 - 12 = 18 cm

Solution 6:

Let O be the centre of the circle and AB and CD be the two parallel chords of length 30 cm and 16 cm respectively.

Drop OE and OF perpendicular on AB and CD from the centre O.



$OE \perp AB$ and $OF \perp CD$

$\therefore$ OE bisects AB and OF bisects CD

(Perpendicular drawn from the centre of a circle to a chord bisects it)

$\Rightarrow \quad AE = \frac{30}{2} = 15\text{ cm}\ ;\ \ CF = \frac{16}{2} = 8\text{ cm}$

In right ΔOAE,

$$OA^2 = OE^2 + AE^2$$

$$\Rightarrow \quad OE^2 = OA^2 - AE^2 = (17)^2 - (15)^2 = 64$$

$$\therefore \quad OE = 8\text{ cm}$$

In right ΔOCF,

$$OC^2 = OF^2 + CF^2$$

$$\Rightarrow \quad OF^2 = OC^2 - CF^2 = (17)^2 - (8)^2 = 225$$

$$\therefore \quad OF = 15\text{ cm}$$

(i) The chords are on the opposite sides of the centre:

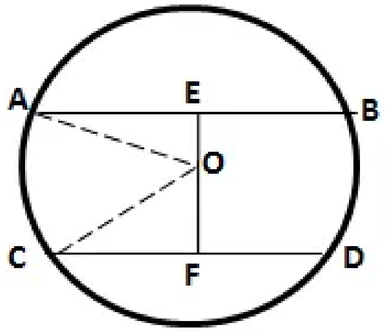
$$\therefore \quad EF = EO + OF = (8+15) = 23\text{ cm}$$

(ii) The chords are on the same side of the centre:

$$\therefore \quad EF = OF - OE = (15 - 8) = 7\text{ cm}$$

Solution 7:

Since the distance between the chords is greater than the radius of the circle (15 cm), so the chords will be on the opposite sides of the centre.



Let O be the centre of the circle and AB and CD be the two parallel chords such that AB = 24 cm.
Let length of CD be 2x cm.
Drop OE and OF perpendicular on AB and CD from the centre O.

$OE \perp AB$ and $OF \perp CD$

$\therefore$ OE bisects AB and OF bisects CD

(Perpendicular drawn from the centre of a circle to a chord bisects it)

$\Rightarrow \quad AE = \frac{24}{2} = 12 \text{ cm}; \quad CF = \frac{2x}{2} = x \text{ cm}$

In right ΔOAE,

$OA^2 = OE^2 + AE^2$

$\Rightarrow \quad OE^2 = OA^2 - AE^2 = (15)^2 - (12)^2 = 81$

$\therefore \quad OE = 9 \text{ cm}$

$\therefore \quad OF = EF - OE = (21 - 9) = 12 \text{ cm}$

In right ΔOCF,

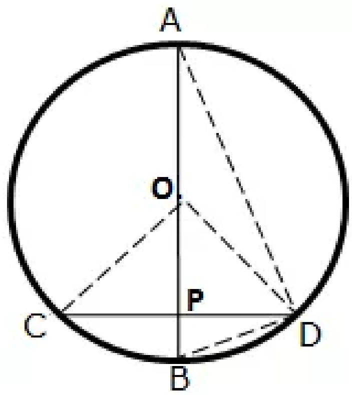
$OC^2 = OF^2 + CF^2$

$\Rightarrow \quad x^2 = OC^2 - OF^2 = (15)^2 - (12)^2 = 81$

$\therefore \quad x = 9 \text{ cm}$

Hence, length of chord $CD = 2x = 2 \times 9 = 18 \text{ cm}$

Solution 8:



(i) $OP \perp CD$

$\therefore$ OP bisects CD

(Perpendicular drawn from the centre of a circle to a chord bisects it)

$\Rightarrow \quad CP = \frac{CD}{2}$

In right ΔOPC,

$$OC^2 = OP^2 + CP^2$$

$$\Rightarrow \quad CP^2 = OC^2 - OP^2 = (15)^2 - (9)^2 = 144$$

$\therefore \quad CP = 12\,cm$

$\therefore \quad CD = 12 \times 2 = 24\ cm$

(ii) Join BD.

$\therefore \quad BP = OB - OP = 15 - 9 = 6\,cm$

In right ΔBPD,

$$BD^2 = BP^2 + PD^2$$

$$= (6)^2 + (12)^2 = 180$$

In ΔADB, $\angle ADB = 90^o$

(Angle in a semicircle is a right angle)

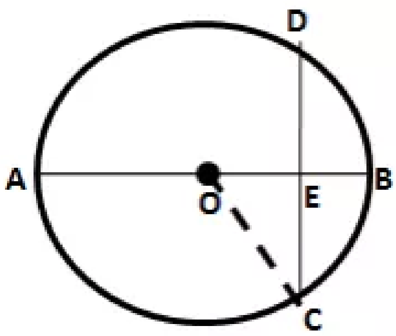
$\therefore \quad AB^2 = AD^2 + BD^2$

$\Rightarrow \quad AD^2 = AB^2 - BD^2 = (30)^2 - 180 = 720$

$\therefore \quad AD = \sqrt{720} = 26.83\,cm$

(iii) Also, $BC = BD = \sqrt{180} = 13.42\ cm$

Solution 9:



Let the radius of the circle be r cm.

$\therefore \quad OE = OB - EB = r - 4$

Join OC.

In right ΔOEC,

$$OC^2 = OE^2 + CE^2$$

$$\Rightarrow r^2 = (r-4)^2 + (8)^2$$

$$\Rightarrow r^2 = r^2 - 8r + 16 + 64$$

$$\Rightarrow 8r = 80$$

$$\therefore r = 10 \text{ cm}$$

Hence, radius of the circle is 10 cm.

Solution 10:

(i) AB is the chord of the circle and OM is perpendicular to AB.

So, AM = MB = 12 cm (Since ⊥ bisects the chord)

In right ΔOMA,

$OA^2 = OM^2 + AM^2$

$\Rightarrow OA^2 = 5^2 + 12^2$

$\Rightarrow OA = 13$ cm

So, radius of the circle is 13 cm.

(ii) So, OA = OC = 13 cm (radii of the same circle)

In right ΔONC,

$NC^2 = OC^2 - ON^2$

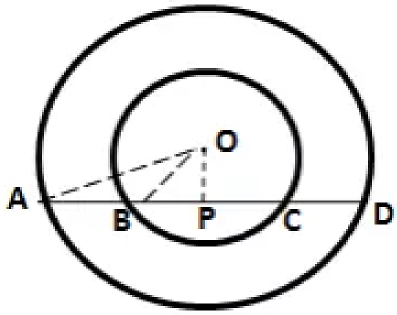
$\Rightarrow NC^2 = 13^2 - 12^2$

$\Rightarrow NC = 5$ cm

So, CD = 2NC = 10 cm

Exercise 17(B)

Solution 1:



Drop $OP \perp AD$

$\therefore$ OP bisects AD

(Perpendicular drawn from the centre of a circle to a chord bisects it)

$\Rightarrow \quad AP = PD \qquad ----(i)$

Now, BC is a chord for the inner circle and $OP \perp BC$

$\therefore$ OP bisects BC

(Perpendicular drawn from the centre of a circle to a chord bisects it)

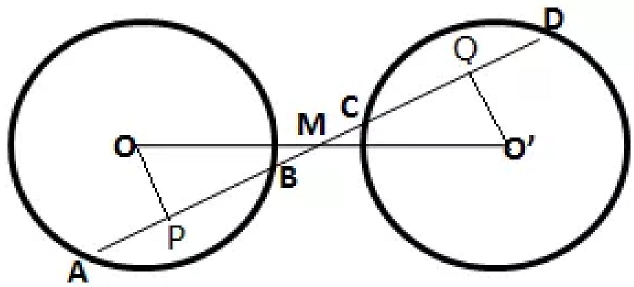
$\Rightarrow \quad BP = PC \qquad ----(ii)$

Subtracting (ii) from (i),

$AP - BP = PD - PC$

$\Rightarrow \quad AB = CD$

Solution 2:



Given: A straight line Ad intersects two circles of equal radii at A, B, C and D.

The line joining the centres OO' intersect AD at M and M is the midpoint of OO'.

To prove: AB = CD.

Construction: From O, draw OP ⊥ AB and from O', draw O'Q ⊥ CD.

Proof:

In ΔOMP and ΔO'MQ,

∠OMP = ∠O'MQ (Vertically opposite angles)

∠OPM = ∠O'QM $\left(\text{each} = 90^0\right)$

OM = O'M (Given)

By Angle-Angle-Side criterion of congruence,

∴ ΔOMP ≅ ΔO'MQ, (by AAS)

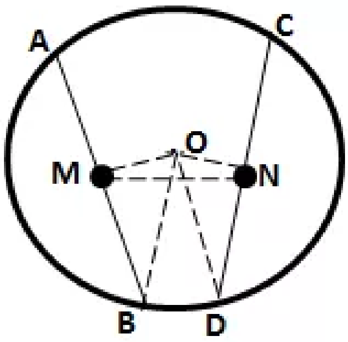
The corresponding parts of the congruent triangles are congruent

∴ OP = O'Q (c.p.c.t)

We know that two chords of a circle or equal circles which are equidistant from the centre are equal.

∴ AB = CD

Solution 3:



Drop $OM \perp AB$ and $ON \perp CD$

$\therefore$ OM bisects AB and ON bisects CD.

(Perpendicular drawn from the centre of a circle to a chord bisects it)

$\Rightarrow \quad BM = \frac{1}{2}AB = \frac{1}{2}CD = DN \qquad ----(1)$

Applying Pythagoras theorem,

$$OM^2 = OB^2 - BM^2$$
$$= OD^2 - DN^2 \qquad \text{(by (1))}$$
$$= ON^2$$

$\therefore \quad OM = ON$

$\Rightarrow \quad \angle OMN = \angle ONM \qquad ----(2)$

(Angles opp to equal sides are equal)

(i) $\quad \angle OMB = \angle OND \qquad (\text{both } 90^0)$

Subtracting (2) from above,

$\angle BMN = \angle DNM$

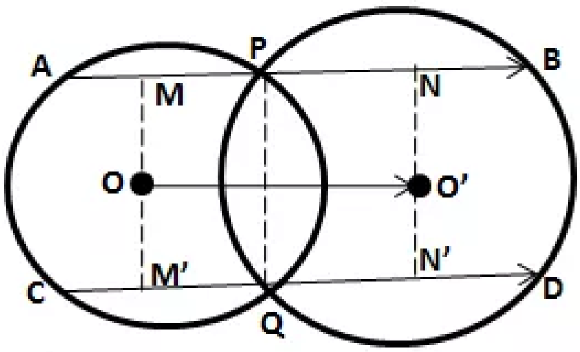
(ii) $\quad \angle OMA = \angle ONC \qquad (\text{both } 90^0)$

Adding (2) to above,

$\angle AMN = \angle CNM$

Solution 4:

Drop OM and O'N perpendicular on AB and OM' and O'N' perpendicular on CD.



$\therefore$ OM, O'N, OM' and O'N' bisect AP, PB, CQ and QD respectively

(Perpendicular drawn from the centre of a circle to a chord bisects it)

$$\therefore \quad MP = \frac{1}{2}AP,\ PN = \frac{1}{2}BP,\ M'Q = \frac{1}{2}CQ,\ QN' = \frac{1}{2}QD$$

$$\text{Now,} \quad OO' = MN = MP + PN = \frac{1}{2}(AP + BP) = \frac{1}{2}AB \text{---(i)}$$

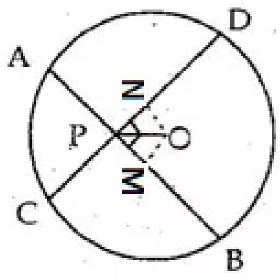
$$\text{and} \quad OO' = M'N' = M'Q + QN' = \frac{1}{2}(CQ + QD) = \frac{1}{2}CD \quad \text{---(ii)}$$

By (i) and (ii),

$$AB = CD$$

Solution 5:

Drop OM and ON perpendicular on AB and CD.
Join OP, OB and OD.



$\therefore$ OM and ON bisect AB and CD respectively

(Perpendicular drawn from the centre of a circle to a chord bisects it)

$\therefore \quad MB = \frac{1}{2}AB = \frac{1}{2}CD = ND \quad ---(i)$

In rtΔOMB, $\quad OM^2 = OB^2 - MB^2 \quad ---(ii)$

In rtΔOND, $\quad ON^2 = OD^2 - ND^2 \quad ---(iii)$

From (i), (ii) and (iii),

$OM = ON$

In ΔOPM and ΔOPN,

$\angle OMP = \angle ONP \quad (\text{both } 90^0)$

$OP = OP \quad$ (Common)

$OM = ON \quad$ (Proved above)

By Right Angle-Hypotenuse-Side criterion of congruence,

$\therefore \quad \Delta OPM \cong \Delta OPN$ (by RHS)

The corresponding parts of the congruent triangles are congruent.

$\therefore \quad PM = PN \quad$ (c.p.c.t)

Adding (i) to both sides,

$MB + PM = ND + PN$

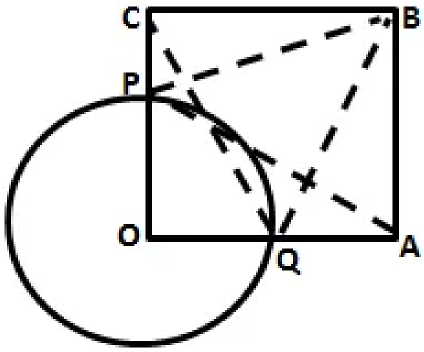
$\Rightarrow \quad BP = DP$

Now, $\quad AB = CD$

$\therefore \quad AB - BP = CD - DP \quad (\because BP = DP)$

$\Rightarrow \quad AP = CP$

Solution 6:



(i)

In ΔOPA and ΔOQC,

$OP = OQ$ (radii of same circle)

$\angle AOP = \angle COQ$ (both 90^0)

$OA = OC$ (sides of the square)

By Side – Angle – Side criterion of congruence,

$\therefore \quad \Delta OPA \cong \Delta OQC$ (by SAS)

(ii)

Now, $OP = OQ$ (radii)

and $OC = OA$ (sides of the square)

$\therefore \quad OC - OP = OA - OQ$

$\Rightarrow \quad CP = AQ$ ----(1)

In ΔBPC and ΔBQA,

$BC = BA$ (sides of the square)

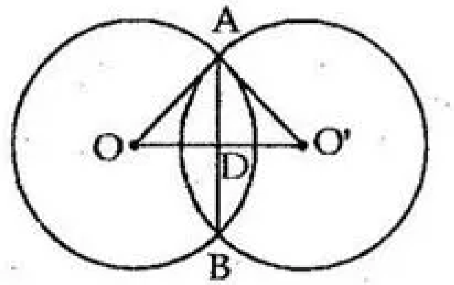
$\angle PCB = \angle QAB$ (both 90^0)

$PC = QA$ (by (1))

By Side – Angle – Side criterion of congruence,

$\therefore \quad \Delta BPC \cong \Delta BQA$ (by SAS)

Solution 7:



$OA = 25\,cm \quad$ and $AB = 30$ cm

$\therefore \quad AD = \frac{1}{2} \times AB = \left(\frac{1}{2} \times 30\right) cm = 15$ cm

Now in right angled Δ ADO,

$$OA^2 = AD^2 + OD^2$$

$$\Rightarrow \quad OD^2 = OA^2 - OD^2 = 25^2 - 15^2$$

$$= 625 - 225 = 400$$

$$\therefore \quad OD = \sqrt{400} = 20 \text{ cm}$$

Again, we have $O'A = 17$ cm.

In right angle Δ ADO'

$$O'A^2 = AD^2 + O'D^2$$

$$\Rightarrow \quad O'D^2 = O'A^2 - AD^2 = 17^2 - 15^2$$

$$= 289 - 225 = 64$$

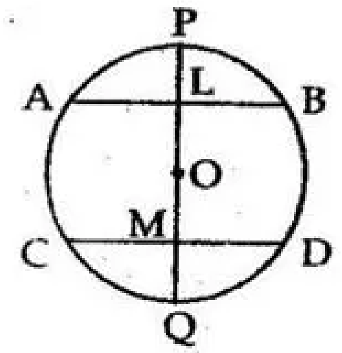
$$\therefore \quad O'D = 8 \text{ cm}$$

$$\therefore \quad OO' = (OD + O'D)$$

$$= (20 + 8) = 28 \text{ cm}$$

$\therefore$ the dis tan ce between their centres is 28 cm.

Solution 8:



Given: AB and CD are the two chords of a circle with centre O.
L and M are the midpoints of AB and CD and O lies in the line joining ML

To Prove : AB || CD.

Proof: AB and CD are two chords of a circle with centre O.

Line LOM bisects them at L and M.

Then, $\quad OL \perp AB$

and, $\quad OM \perp CD$

$\therefore \quad \angle ALM = \angle LMD = 90^0$

But they are alternate angles

$\therefore \quad AB \parallel CD.$

Solution 9:

In the circle with centre Q, $QO \perp AD$

$\therefore \quad OA = OD \quad ----(1)$

(Perpendicular drawn from the centre of a circle to a chord bisects it)

In the circle with centre P, $PO \perp BC$

$\therefore \quad OB = OC \quad ----(2)$

(Perpendicular drawn from the centre of a circle to a chord bisects it)

(i)

(1) – (2) gives,

$AB = CD \quad ----(3)$

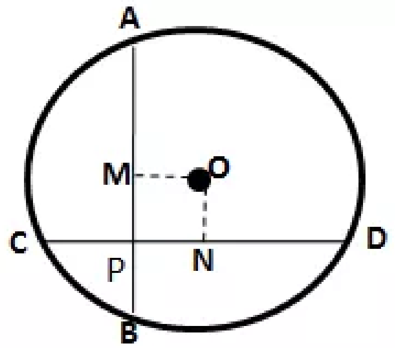
(ii)

Adding BC to both sides of equation (3)

$AB + BC = CD + BC$

$\Rightarrow \quad AC = BD$

Solution 10:



Clearly, all the angles of OMPN are 90^0.

$OM \perp AB$ and $ON \perp CD$

$\therefore \quad BM = \frac{1}{2}AB = \frac{1}{2}CD = CN \quad ----(i)$

(Perpendicular drawn from the centre of a circle to a chord bisects it)

As the two equal chords AB and CD intersect at point P inside the circle,

$\therefore \quad AP = DP$ and $CP = BP \quad ----(ii)$

Now, $CN - CP = BM - BP \quad$ (by (i) and (ii))

$\Rightarrow \quad PN = MP$

$\therefore \quad$ Quadrilateral OMPN is a square.

Exercise 17(C)

Solution 1:

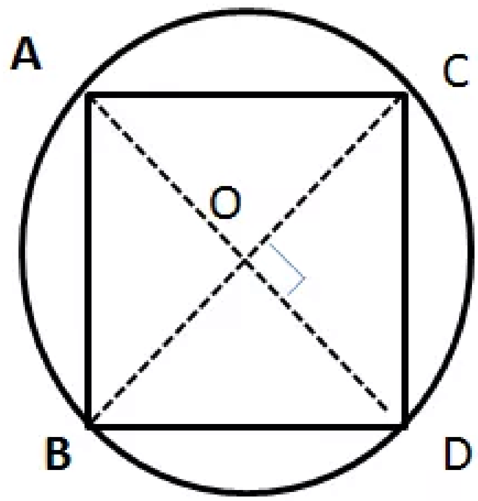
In the given figure, ΔABC is an equilateral triangle.
Hence all the three angles of the triangle will be equal to 60°.
i.e. $\angle A = \angle B = \angle C = 60^\circ$
As the triangle is an equilateral triangle,
BO and CO will be the angle bisectors of $\angle B$ and $\angle C$ respectively.

$$\text{Hence } \angle OBC = \frac{\angle ABC}{2}$$
$$= 30^\circ$$

and as given in the figure we can see that OB and OC are the radii of the given circle
Hence they are of equal length.
The ΔOBC is an isosceles triangle with OB = OC
In ΔOBC, $\angle OBC = \angle OCB$ as they are angles opposite to the two equal sides of an isosceles triangle.
Hence, $\angle OBC = 30^\circ$ and $\angle OCB = 30^\circ$
Since the sum of all the angles of a triangle is 180°
Hence in triangle OBC, $\angle OCB + \angle OBC + \angle BOC = 180^\circ$
$30^\circ + 30^\circ + \angle BOC = 180^\circ$
$60^\circ + \angle BOC = 180^\circ$
$\angle BOC = 180^\circ - 60^\circ$
$\angle BOC = 120^\circ$
Hence $\angle BOC = 120^\circ$ and $\angle OBC = 30^\circ$

Solution 2:

In the given figure we can extend the staight line OB to BD and CO to CA
Then we get the diagonals of the square which intersect each other at 90° by the property of Square.



From the above statement we can see that

$\angle COD = 90°$.

The sum of the angle $\angle BOC$ and $\angle OCD$ is 180° as BD is a straight line.

Hence $\angle BOC + \angle OCD = \angle BOD = 180°$

$\angle BOC + 90° = 180°$

$\angle BOC = 180° - 90°$

$\angle BOC = 90°$

We can see that the ΔOCB is an isosceles triangle with sides OB and OC of equal length as they are the radii of the same circle.

In ΔOCB , $\angle OBC = \angle OCB$ as they are oppsite angles to the two equal sides of an isosceles triangle.

Sum of all the angles of a triangle is 180°

so, $\angle OBC + \angle OCB + \angle BOC = 180°$

$\angle OBC + \angle OBC + 90° = 180°$ as, $\angle OBC = \angle OCB$

$2\angle OBC = 180° - 90°$

$2\angle OBC = 90°$

$\angle OBC = 45°$

as $\angle OBC = \angle OCB$ So,

$\angle OBC = \angle OCB = 45°$

Yes BD is the diameter of the circle.

Solution 3:

As given that AB is the side of a pentagon the angle subtended by each arm of the pentagon at

the centre of the circle is $= \frac{360°}{5} = 72°$

Thus angle $\angle AOB = 72°$

Similarly as BC is the side of a hexagon hence the angle subtended

by BC at the centre is $= \frac{360°}{6}$

i.e. 60°

$\angle BOC = 60°$

Now $\angle AOC = \angle AOB + \angle BOC = 72° + 60° = 132°$

The triangle thus formed , ΔAOB is an isosceles triangle

with OA = OB as they are radii of the same circle.

Thus $\angle OBA = \angle BAO$ as they are opposite angles of equal sides

of an isosceles triangle.

The sum of all the angles of a triangle is 180°

so, $\angle AOB + \angle OBA + \angle BAO = 180°$

$2\angle OBA + 72° = 180°$ as ,$\angle OBA = \angle BAO$

$2\angle OBA = 180° - 72°$

$2\angle OBA = 108°$

$\angle OBA = 54°$

as $\angle OBA = \angle BAO$ So,

$\angle OBA = \angle BAO = 54°$

The triangle thus formed , ΔBOC is an isosceles triangle

with OB = OC as they are radii of the same circle.

Thus $\angle OBC = \angle OCB$ as they are opposite angles of equal sides

of an isosceles triangle.

The sum of all the angles of a triangle is 180°

so, $\angle BOC + \angle OBC + \angle OCB = 180°$

$2\angle OBC + 60° = 180°$ as ,$\angle OBC = \angle OCB$

$2\angle OBC = 180° - 60°$

$2\angle OBC = 120°$

$\angle OBC = 60°$

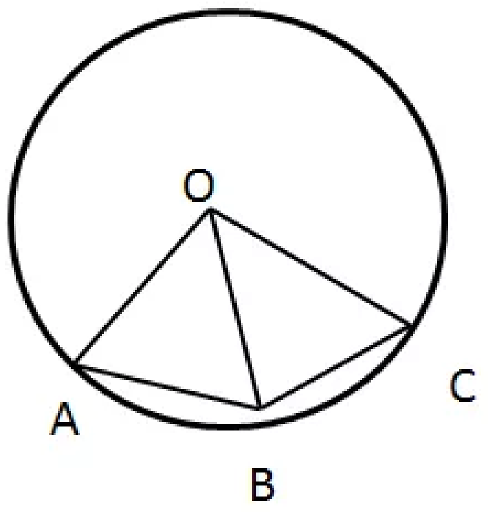
as $\angle OBC = \angle OCB$

So, $\angle OBC = \angle OCB = 60°$

$\angle ABC = \angle OBA + \angle OBC = 54° + 60° = 114°$

Solution 4:

We know that the arc of equal lengths subtend equal angles at the centre.



hence $\angle AOB = \angle BOC = 48^\circ$

Then $\angle AOC = \angle AOB + \angle BOC = 48^\circ + 48^\circ = 96^\circ$

The triangle thus formed , ΔBOC is an isosceles triangle with OB = OC as they are radii of the same circle.

Thus $\angle OBC = \angle OCB$ as they are opposite angles of equal sides of an isosceles triangle.

The sum of all the angles of a triangle is 180°

so, $\angle BOC + \angle OBC + \angle OCB = 180^\circ$

$2\angle OBC + 48^\circ = 180^\circ$ as $\angle OBC = \angle OCB$

$2\angle OBC = 180^\circ - 48^\circ$

$2\angle OBC = 132^\circ$

$\angle OBC = 66^\circ$

as $\angle OBC = \angle OCB$

So, $\angle OBC = \angle OCB = 66^\circ$

The triangle thus formed , ΔAOC is an isosceles triangle with OA = OC as they are radii of the same circle.

Thus $\angle OAC = \angle OCA$ as they are opposite angles of equal sides of an isosceles triangle.

The sum of all the angles of a triangle is 180°

so, $\angle COA + \angle OAC + \angle OCA = 180^\circ$

$2\angle OAC + 96^\circ = 180^\circ$ as ,$\angle OAC = \angle OCA$

$2\angle OAC = 180^\circ - 96^\circ$

$2\angle OAC = 84^\circ$

$\angle OAC = 42^\circ$

as $\angle OCA = \angle OAC$

So,$\angle OCA = \angle OAC = 42^\circ$

Solution 5:

We know that for two arcs are in ratio 3:2 then

$\angle AOB : \angle BOC = 3:2$

As give $\angle AOC = 96°$

So, $3x=96$

$x = 32$

Therefore $\angle BOC = 2 \times 32 = 64°$

The triangle thus formed , ΔAOB is an isosceles triangle with OA = OB as they are radii of the same circle.

Thus $\angle OBA = \angle BAO$ as they are opposite angles of equal sides of an isosceles triangle.

The sum of all the angles of a triangle is 180°

so, $\angle AOB+\angle OBA+\angle BAO=180°$

$2\angle OBA + 96° = 180°$ as ,$\angle OBA = \angle BAO$

$2\angle OBA = 180° - 96°$

$2\angle OBA = 84°$

$\angle OBA = 42°$

as $\angle OBA = \angle BAO$ So,

$\angle OBA = \angle BAO = 42°$

The triangle thus formed , ΔBOC is an isosceles triangle with OB = OC as they are radii of the same circle.

Thus $\angle OBC = \angle OCB$ as they are opposite angles of equal sides of an isosceles triangle.

The sum of all the angles of a triangle is 180°

so, $\angle BOC+\angle OBC+\angle OCB = 180°$

$2\angle OBC+64° = 180°$ as ,$\angle OBC = \angle OCB$

$2\angle OBC = 180° - 64°$

$2\angle OBC= 116°$

$\angle OBC= 58°$

as $\angle OBC = \angle OCB$ So,

$\angle OBC = \angle OCB = 58°$

$\angle ABC = \angle BOA + \angle OBC = 42° + 58° = 100°$

Solution 6:

Since arc AB and BC are equal

so, $\angle AOB = \angle BOC = 50^\circ$

Now

$\angle AOC = \angle AOB + \angle BOC = 50^\circ + 50^\circ = 100^\circ$

As arc AB, arc BC and arc CD so,

$\angle AOB = \angle BOC = \angle COD = 50^\circ$

$\angle AOD = \angle AOB + \angle BOC + \angle COD = 50^\circ + 50^\circ + 50^\circ = 150^\circ$

Now, $\angle BOD = \angle BOC + \angle BOD$

$\angle BOD = 50^\circ + 50^\circ$

$\angle BOD = 100^\circ$

The triangle thus formed , ΔAOC is an isosceles triangle with OA = OC as they are radii of the same circle.

Thus $\angle OAC = \angle OCA$ as they are opposite angles of equal sides of an isosceles triangle.

The sum of all the angles of a triangle is 180°

so, $\angle AOC + \angle OAC + \angle OCA = 180^\circ$

$2\angle OAC + 100^\circ = 180^\circ$ as ,$\angle OAC = \angle OCA$

$2\angle OAC = 180^\circ - 100^\circ$

$2\angle OAC = 80^\circ$

$\angle OAC = 40^\circ$

as $\angle OCA = \angle OAC$ So,

$\angle OCA = \angle OAC = 40^\circ$

The triangle thus formed , ΔAOD is an isosceles triangle with OA = OD as they are radii of the same circle.

Thus $\angle OAD = \angle ODA$ as they are opposite angles of equal sides of an isosceles triangle.

The sum of all the angles of a triangle is 180°

so, $\angle AOD + \angle OAD + \angle ODA = 180^\circ$

$2\angle ODA + 150^\circ = 180^\circ$ as ,$\angle OAD = \angle ODA$

$2\angle ODA = 180^\circ - 150^\circ$

$2\angle ODA = 30^\circ$

$\angle ODA = 15^0$

as $\angle OAD = \angle ODA$ So,

$\angle OAD = \angle ODA = 15^\circ$

Solution 7:

As AB is the side of a hexagon so the

$\angle AOB = \frac{360^\circ}{6} = 60^\circ$

AC is the side of an eight sided polygon so,

$\angle AOC = \frac{360^\circ}{8} = 45^\circ$

From the given figure we can see that:

$\angle BOC = \angle AOB + \angle AOC = 60^\circ + 45^\circ = 105^\circ$

Again, from the figure we can see that ΔBOC is an isosceles triangle with sides BO = OC as they are the radii of the same circle.

Angles $\angle OBC = \angle OCB$ as they are opposite angles to the equal sides of an isoseles triangle.

Sum of all the angles of a triangle is 180°

$\angle OBC + \angle OCB + \angle BOC = 180^\circ$

$2\angle OBC + 105^\circ = 180^\circ$ as, $\angle OBC = \angle BOC$

$2\angle OBC = 180^\circ - 105^\circ$

$2\angle OBC = 75^\circ$

$\angle OBC = 37.5^\circ = 37^\circ 30'$

As, $\angle OBC = \angle BOC$

$\angle OBC = \angle BOC = 37.5^\circ = 37^\circ 30'$

Solution 8:

We know that when two arcs are in ratio 2:1 then the subtended by them is also in ratio 2:1

As given arc AB is twice the length of arc BC

Therefore, arc AB: arc BC = 2:1

Hence, $\angle AOB : \angle BOC = 2:1$

Now given that $\angle AOB = 100°$

so, $\angle BOC = \frac{1}{2}\angle AOB = \frac{1}{2} \times 100° = 50°$

Now, $\angle AOC = \angle AOB + \angle BOC = 100° + 50° = 150°$

The triangle thus formed, ΔAOC is an isosceles triangle with OA = OC as they are radii of the same circle.

Thus $\angle OAC = \angle OCA$ as they are opposite angles of equal sides of an isosceles triangle.

The sum of all the angles of a triangle is 180°

so, $\angle COA + \angle OAC + \angle OCA = 180°$

$2\angle OAC + 150° = 180°$ as $\angle OAC = \angle OCA$

$2\angle OAC = 180° - 150°$

$2\angle OAC = 30°$

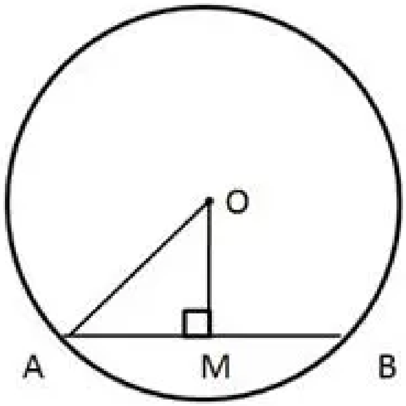
$\angle OAC = 15^0$

as $\angle OCA = \angle OAC$ So,

$\angle OCA = \angle OAC = 15°$

Exercise 17(D)

Solution 1:



To find : OM

Given that AB = 24 cm

Since $OM \perp AB$

$\Rightarrow$ OM bisects AB

So, AM = 12 cm

In right ΔOMA,

$OA^2 = OM^2 + AM^2$

$\Rightarrow OM^2 = OA^2 - AM^2$

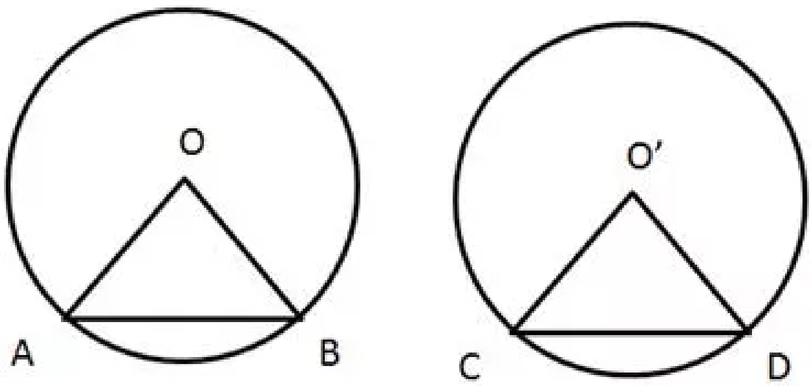
$\Rightarrow OM^2 = 13^2 - 12^2$

$\Rightarrow OM^2 = 25$

$\Rightarrow OM = 5$ cm

Hence, the distance of the chord from the centre is 5 cm.

Solution 2:



Given: AB and CD are two equal chords of congruent circles with centres O and O' respectively.

To prove: $\angle AOB = \angle CO'D$

Proof: In ΔOAB and $\Delta O'CD$,

$OA = O'C$ ($\because$ Radii of congruent circles)

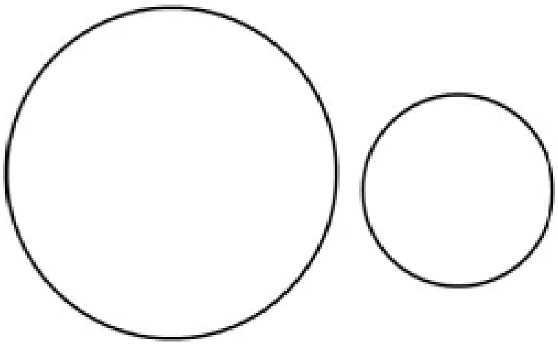
$OB = O'D$ ($\because$ Radii of congruent circles)

$AB = CD$ (Given)

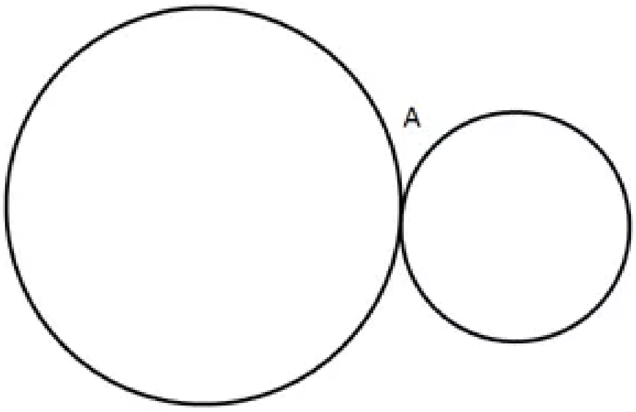
$\Delta OAB \cong \Delta O'CD$ (By SSS congruence criterion)

$\angle AOB = \angle CO'D$ (cpct)

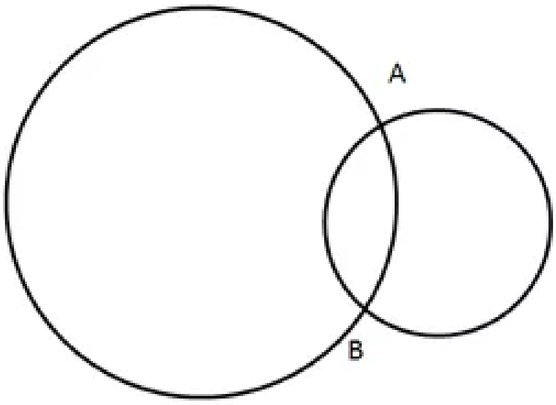
Solution 3:



No point of intersection



One point of intersection

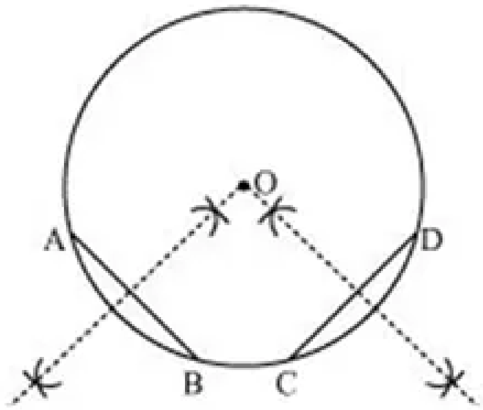


Two points of intersection

So, the circle can have 0, 1 or 2 points in common.

The maximum number of common points is 2.

Solution 4:

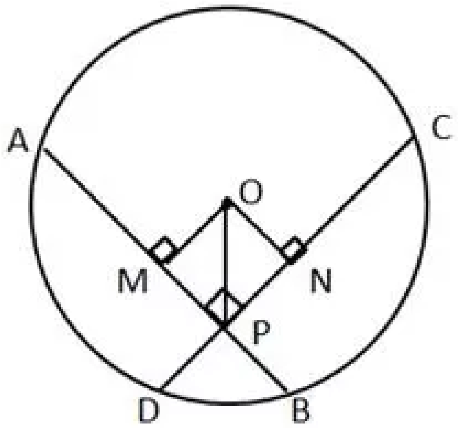


To draw the centre of a given circle:

1. Draw the circle.
2. Take any two different chords AB and CD of this circle and draw perpendicular bisectors of these chords.
3. Let these perpendicular bisectors meet at point O.

So, O will be the centre of the given circle.

Solution 5:



In ΔOMP and ΔONP,

$OP = OP$ (common side)

$\angle OMP = \angle ONP$ (both are right angles)

$OM = OM$ (side both the chords are equal, so the distance of the chords from the centre are also equal)

$\Delta OMP \cong \Delta ONP$ (RHS congruence criterion)

$\Rightarrow MP = PN$ (cpct)

....(a)

(i) Since $AB = CD$ (given)

$\Rightarrow AM = CN$ ($\perp$ drawn from the centre to the chord bisects the chord)

$\Rightarrow AM + MP = CN + NP$ (from (a))

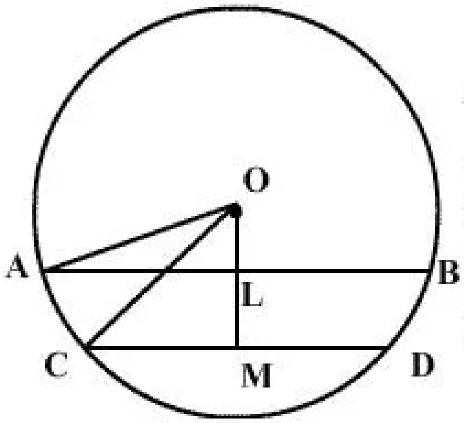
$\Rightarrow AP = CP$...(b)

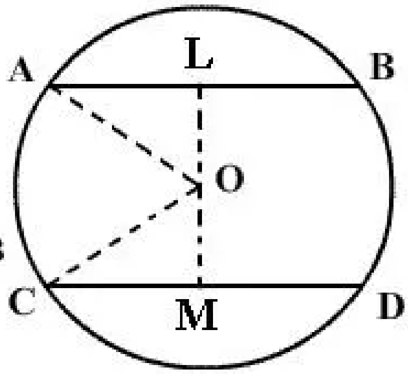
(ii) Since $AB = CD$

$\Rightarrow AP + BP = CP + DP$

$\Rightarrow BP = DP$ (from (b))

Solution 6:





Given that AB = 16 cm and CD = 12 cm

So, AL = 8 cm and CM = 6 cm (⊥ from the centre to the chord bisects the chord)

In right triangles OLA and OMC,

By Pythagoras theorem,

$OA^2 = OL^2 + AL^2$ and $OC^2 = OM^2 + CM^2$

$\Rightarrow 10^2 = OL^2 + 8^2$ and $10^2 = OM^2 + 6^2$

$\Rightarrow OL^2 = 100 - 64$ and $OM^2 = 100 - 36$

$\Rightarrow OL^2 = 36$ and $OM^2 = 64$

$\Rightarrow OL = 6$ cm and $OM = 8$ cm

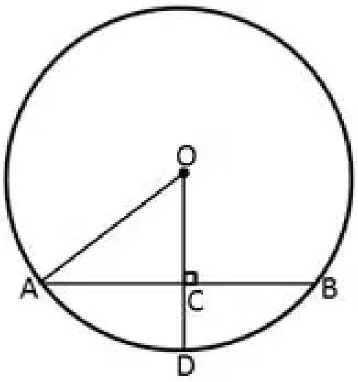
(i) In the first case, distance between AB and CD is

$LM = OM - OL = 8 - 6 = 2$ cm

(ii) In the second case, distance between AB and CD is

$LM = OM + OL = 8 + 6 = 14$ cm

Solution 7:



To find : CD

Given AB = 32 cm

$\Rightarrow$ AC = 16 cm (Since $\perp$ drawn from the centre to the chord, bisects the chord)

In right ΔOCA,

$OA^2 = OC^2 + AC^2$ (By Pythagoras theorem)

$\Rightarrow OC^2 = OA^2 - AC^2$

$\Rightarrow OC^2 = 20^2 - 16^2$

$\Rightarrow OC^2 = 144$

$\Rightarrow OC = 12$ cm

Since OD = 20 cm and OC = 12 cm

$\Rightarrow CD = OD - OC = 20 - 12 = 8$ cm

Solution 8:

It is given in the question that point
P is the midpoint of the chord AB and and point Q is the midpoint of the chord CD .

$\Rightarrow \angle APO = 90^\circ$ $\left(\begin{array}{l}\text{as the straight line drawn from the centre of a circle} \\ \text{to bisect a chord, which is not a diameter, is at the} \\ \text{right angle to the chord}\end{array}\right)$

As chords AB and CD are equal therefore they are equidistant from the centre i.e. PO = OQ (∵ Equal chords of a circle are equidistant from the centre)

Now the ΔPOQ is an isosceles triangle with OP = OQ as its two equal sides

Therefore $\angle OPQ = \angle PQO$, as they are opposite angles to the equal sides of an isosceles triangle.

Sum of all the angles of a triangle is 180^0

$\Rightarrow \angle POQ + \angle OPQ + \angle PQO = 180^\circ$

$\Rightarrow \angle OPQ + \angle POQ + 150^\circ = 180^\circ$ $[\text{Given: } \angle POQ = 150^\circ]$

$\Rightarrow 2\angle OPQ = 180^\circ - 150^\circ$ $[\text{As, } \angle OPQ = \angle PQO]$

$\Rightarrow 2\angle OPQ = 30^\circ$

$\Rightarrow \angle OPQ = 15^\circ$

As $\angle APO = 90^\circ$

$\Rightarrow \angle APQ + \angle OPQ = 90^\circ$

$\Rightarrow \angle APQ = 90^\circ - 15^\circ$ $[\text{As, } \angle OPQ = 15^\circ]$

$\Rightarrow \angle APQ = 75^\circ$

Solution 9:

Given :

1. AOC is the diameter

2. Arc AXB $= \frac{1}{2}$ Arc BYC

From Arc AXB $= \frac{1}{2}$ Arc BYC we can see that

Arc AXB : Arc BYC = 1:2

$\Rightarrow \angle BOA : \angle BOC = 1:2$

Since AOC is the diameter of the circle hence,

$\angle AOC = 180^\circ$

Now,

Assume that $\angle BOA = x^\circ$ and $\angle BOC = 2x^\circ$

$\angle AOC = \angle BOA + \angle BOC = 180^\circ$

$\Rightarrow x + 2x = 180$

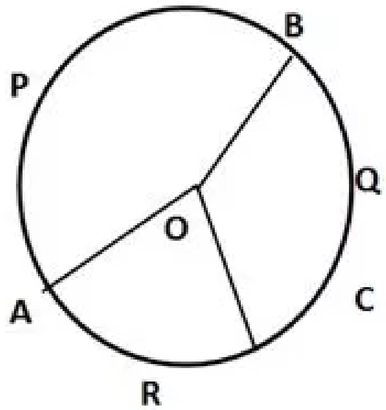
$\Rightarrow 3x = 180$

$\Rightarrow x = 60$

Hence $\angle BOA = 60^\circ$ and $\angle BOC = 120^\circ$

Solution 10:

From the given conditions given in the question
we can draw the circle with arc APB, arc BQC and arc CRA



The given equation is

$$\frac{\text{Arc APB}}{2} = \frac{\text{Arc BQC}}{3} = \frac{\text{Arc CRA}}{4}$$

let

$$\frac{\text{Arc APB}}{2} = \frac{\text{Arc BQC}}{3} = \frac{\text{Arc CRA}}{4} = k\text{(Say)}$$

then Arc APB = 2k, Arc BQC = 3k, Arc CRA = 4k

or

Arc APB : Arc BQC : Arc CRA = 2 : 3 : 4

$\Rightarrow \angle AOB : \angle BOC : \angle AOC = 2 : 3 : 4$

and therefore

and $\angle AOB = (2k)^\circ$, $\angle BOC = (3k)^\circ$ and $\angle AOC = (4k)^\circ$

Now,

Angle in a circle is 360°

So, $2k + 3k + 4k = 360$

$\Rightarrow 9k = 360$

$\Rightarrow k = 40$

Hence

$\angle BOC = 3 \times 40 = 120^\circ$